American University of Sharjah

Department of Mathematics and Statistics

MTH 203 Sample Exam II, 2009-2012

1. [10pts] Find a value of c such that the function

$$f(x,y) = \frac{x^3}{3} + \frac{cy^2}{2} + xy$$

has a local maximum somewhere. Justify your answer using the second derivative test.

2. [10pts] Consider a metal plate which is a disk of radius $\sqrt{2}$ centered at $(4,0)$ in the xy-plane. Note that the equation of the corresponding circle is $(x-4)^2 + y^2 = 2$. The temperature of the disk is given by $T(x,y) = \ln(x+y)$.

   Find the minimum and maximum temperature on the disk.

3. [10pts] Compute the following integral by reversing the order of integration.

$$\int_0^2 \int_{x^2}^4 \frac{1}{1+y^{3/2}}\, dy\, dx$$

4. [10pts] Write a double integral in **polar coordinates** that equals the surface area of the portion of $x^2+y^2+z^2 = 9$ that lies between $z=1$ and $z=2$. You do NOT have to evaluate the integral.

5. [10pts]Write an integral in spherical coordinates for the mass of the region that lies below $x^2+y^2+z^2 = 4$ and above $z=1$ and has density $\rho(x,y,z) = z$. *(Note that ρ is used for denisty and radius).* You do NOT have to evaluate the integral.

6. [10pts] Write a triple integral that represent the volume of the region that lies above $z = \sqrt{x^2+y^2}$, below $z = 2 + \sqrt{x^2+y^2}$ and inside $z = x^2+y^2$. You do NOT have to evaluate the integral.

1. Use Lagrange multipliers to find the points on the circle $(x-3)^2 + (y-4)^2 = 1$ that are closest and farthest from the origin.

2. For the iterated integral

$$\int_0^2 \int_{\sqrt{y/2}}^1 y\, e^{x^5} dx\, dy$$

Graph the region of integration, change the order and compute **(be careful.)**

3. Find the mass of the lamina in the first quadrant bounded by the circle $x^2 + y^2 = 2$ and the line $y = x$ given that the density function $\rho(x, y) = \dfrac{x}{y}$. Sketch the lamina. [Hint: Use polar coordinates.]

4. Write the integral in cylindrical coodinates **(do not evaluate).**

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{0}^{x} (x^2 + y^2)\, dz\, dx\, dy$$

5. Set up an iterated integral that evaluates the volume of the region in the first octant bounded by the surfaces $2y^2 + z^2 = 8$ and $x + y = 2$.**(do not evaluate).**

6. Let E be the region in the first octant in space, inside the sphere $x^2 + y^2 + z^2 = 9$ and below the cone $3z^2 = x^2 + y^2$.

   (a) Convert the equation of the cone to spherical form.

   (b) Find the volume of the solid.

Q1. (20%) Let $f(x, y) = xy^2 + y^2 - \dfrac{x^2}{2} - 2x + 3$. Find all critical points of $f(x, y)$

and classify each critical point as a local minimum, local maximum or saddle point.

Q2. (16%) Find the max and min values of $f(x, y) = x^2 y$ subject to the constraint $x^2 + 2y^2 = 6$. b) What is the max and min on $x^2 + 2y^2 \leq 6$

Q3. (12%) Sketch the region and change the order of integration for $\int_{-2}^{1} \int_{2y^2-1}^{3-2y} f(x, y) dx dy$

Q4. (12%) Consider the integral $\int_0^{1/2} int_{x/\sqrt{3}}^{\sqrt{2x-x^2}} \dfrac{1}{1 + x^2 + y^2} dy dx$

(a) Sketch the region of integration

(b) Change the above integral to polar coordinates. (Do not evaluate the integral).

Q5.(12%) Set up a double integral to find the area of the surface S of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 9$.

Q6. (12%) Set up a triple integral in rectangular coordinates that represents the volume of the solid enclosed by the cylinder $x^2 + y^2 = 9$ and the planes $y + z = 5$ and $z = 1$. (Do not evaluate the integral)

Q7(3%)a) An equation of a surface in spherical coordinates is given by $\rho = \sin(\varphi)(2\sin(\theta) - \cos(\theta))$. Express the equation in rectangular coordinates and describe the surface.

(10%)b) Set up triple integral in spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{\dfrac{x^2+y^2}{3}}$ and below the sphere $x^2 + y^2 + z^2 = 4z$.(Do not evaluate the integral).

(7%) c) If the the density at the point (x, y, z) is $\rho(x, y, z) = x + z$. Set up an triple integral in cylinderical coordinates which gives the mass of D.(Do not evaluate the integral).